Math Virtual Learning

AP Statistics

Statistical Power

April 14, 2020

Inspiring Greatness
INDEPENDENCE SCHOOL DISTRICT

Lesson: April 14, 2020

Objective/Learning Target:
Students will gain a basic understanding of what the power of a statistical test is representing.

A powerful problem

This activity requires the use of a graphing calculator or similar web app.

It might be easiest to download the PDF file and fill in the blanks as you proceed.

Try your best to come up with answers to each part. When finished, check your answers with the answers provided at the end of this presentation.

A Powerful problem

Consider the scenario in which a cereal company claims that 20% of all its cereal boxes contain a voucher for a free DVD rental. A group of students believes the company is cheating and the proportion of all boxes with the vouchers is less than 0.20. They decide to collect some data to perform a test of significance with the following hypotheses.

$$H_0: p = 0.20$$
$$H_A: p < 0.20$$

where p = the proportion of all boxes with the voucher

They collect a random sample of 65 boxes and find 11 boxes with the voucher. Using a One Proportion *z*-test, the students calculate a *p*-value = 0.27 and conclude that they do not have enough evidence to say that the proportion of all boxes is less than 0.20. Although the company may be cheating its customers, the students do not have convincing evidence that this is the case.

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1-PropZTest
 prop<.2
 z=⁻.6201736729
 p=.2675716664
 p̂=.1692307692
 n=65
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PART I: WILL THE STUDENTS UNCOVER CORPORATE WRONGDOING?

The question this handout addresses is the following.

> If the company is in fact cheating its customers,
> how likely would it be for a test based on 65 boxes to catch the company?

1. Suppose the students used a significance level of $\alpha = .05$ in conducting their test. Explain what this significance level represents and how it affects the decision they made.

2. The students found 11 out of 65 boxes with vouchers and did not conclude the company was cheating. How many boxes with vouchers out of 65 would they have needed to find in order to conclude that the company is cheating? Use trial and error with One Proportion z-test on your calculator to find the range of number of voucher boxes that would lead to a conclusion of corporate wrongdoing.

The assumption in this handout is the company is cheating, and the question is how likely would it be for the students' 65 box test with $\alpha = .05$ to detect this cheating.

A natural question to ask then is: How badly is the company cheating? Pretend the company's proportion of all boxes with vouchers is really 0.15 ($p = 0.15$). If a 65 box test using $\alpha = .05$ were performed, would the students correctly conclude that the company is cheating ($p < 0.20$)? Let's find out.

3. You will sample 65 cereal boxes from a population in which 15% of all boxes contain a voucher. The calculator command below, which can be found under MATH – PRB, simulates random sampling from this population. Run this command to take a sample of 65.

randBin(65,.15)

In question #2, you should have arrived at the following rule for concluding that the company is cheating. (Recall this rule is based on the significance level of $\alpha = .05$.)

Conclude the company is cheating if you obtain 7 or fewer boxes with vouchers out of 65.

4. In your 65 box trial from question #3, how many boxes with vouchers did you obtain? Based on your result, did you have enough evidence to conclude that the company is cheating?

5. Repeat your simulation from question #3 twenty times and record your results in the table below. (To repeat the command randBin(65, .15) simply press ENTER.)

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Voucher Boxes																				

6. Remember, the assumption in the simulation is the company is cheating—$p = 0.15$. Out of your 20 trials in question #5, in how many of them did you conclude that the company is cheating?

The proportion of 20 trials in which you concluded the company is cheating is your estimate of the probability of the 65 box test with $\alpha = .05$ detecting the company's placement of vouchers in only 15% of its boxes.

7. Conduct some more trials to get a better estimate of the power. Remember the more trials the closer this experiment will represent the truth (Law of Large Numbers). What is the class estimate of the probability of the 65 box test with $\alpha = .05$ detecting the company's placement of vouchers in only 15% of its boxes?

The probability you just calculated is an estimate of the POWER of a One Proportion z-test with $n = 65$ and $\alpha = .05$ against the alternative of $p = 0.15$.

8. Using your result in question #7, comment on the students' ability to detect a company that puts vouchers in only 15% of its boxes by using a 65 box test with $\alpha = .05$.

1000 trials of the simulation from question #3 were conducted using computer software. In 226 of these trials, 7 or fewer boxes with vouchers were found, and thus in 22.6% of the trials it was concluded the company was cheating. So, it is not all that likely for the students' 65 box test using $\alpha = .05$ to detect a company whose proportion of all boxes with vouchers is 0.15!

You have seen what power represents in this scenario. A more general definition of POWER is given below and can be applied to any situation in which a test of significance is performed.

The POWER of a test of significance against a given alternative is the probability that it rejects the null hypothesis.

Answers

1. Suppose the students used a significance level of $\alpha = .05$ in conducting their test. Explain what this significance level represents and how it affects the decision they made.

The significance level represents a "cut-off" point for the *P*-value and guides the students in deciding whether to reject or fail to reject the null hypothesis. If the students received a *P*-value that was less than .05, then their decision would have been to reject the null hypothesis and to conclude that the company is in fact cheating its customers (i.e. $p < 0.20$). If the *P*-value was greater than .05, the decision would have been to fail to reject the null hypothesis—the students would not have evidence of the company cheating.

2. The students found 11 out of 65 boxes with vouchers and did not conclude the company was cheating. How many boxes with vouchers out of 65 would they have needed to find in order to conclude that the company is cheating? Use trial and error with One Proportion z-test on your calculator to find the range of number of voucher boxes that would lead to a conclusion of corporate wrongdoing.

7 or fewer boxes with vouchers out of 65 will give a *P*-value less than .05

(Students by this point in the course should have experience with One Prop z-test on the calculator. The intention of this question is to avoid having to calculate *z*-scores and *P*-values by hand and lessen the computational load. We have the technology, so let's use it to our advantage!)

4. In your 65 box trial from question #3, how many boxes with vouchers did you obtain? Based on your result, did you have enough evidence to conclude that the company is cheating?

I received 16 boxes with vouchers when I did the simulation. Thus, I do not have enough evidence to conclude the company is cheating. You will probably have a different number of boxes. If you got 7 or fewer you would have evidence.

5. Repeat your simulation from question #3 twenty times and record your results in the table below. (To repeat the command randBin(65,.15) simply press ENTER.)

I entered my results in the table below. Yours will probably be different.

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Voucher Boxes	8	12	7	9	8	9	9	9	12	7	5	11	16	9	9	6	2	6	8	6

6. Remember, the assumption in the simulation is the company is cheating—$p = 0.15$. Out of your 20 trials in question #5, in how many of them did you conclude that the company is cheating?

Out of the 20 trials, 7 of the trials obtained 7 or fewer boxes. Thus, in 35% of my 20 trials I would conclude that the company is cheating.

7. Conduct some more trials to get a better estimate of the power. Remember the more trials the closer this experiment will represent the truth (Law of Large Numbers). What is the class estimate of the probability of the 65 box test with $\alpha = .05$ detecting the company's placement of vouchers in only 15% of its boxes?

   (Typically when I combine the class' results I get an estimate of anywhere from 20% to 30% of the trials detecting the company's cheating. This is usually based on a couple hundred trials.)

8. Using your result in question #7, comment on the students' ability to detect a company that puts vouchers in only 15% of its boxes by using a 65 box test with $\alpha = .05$.

If the company is cheating the customers by putting vouchers in only 15% of all the boxes, then there is not that great a chance (20-30%) that the 65 box test of the students will detect this cheating. The 65 box test is not very powerful against the alternative of $p = 0.15$.